Rmk μ_{CZ} is called the Conley-Zehnder index of A. One can use this to define CZ-index of a closed Reeb orbit γ (with a trivialization of the contact hyperplane field along γ.

Finally, let's briefly explain a geometric meaning of μ^{spec}.

(inspired by a geometric approach to define CZ-index, as an intersection counting elaborated in previous semester).

Notation:

$$D := W^{1,2}(S^1, \mathbb{R}^{2n}), \quad H := L^2(S^1, \mathbb{R}^{2n}), \quad T_{ref} : D \to H \text{ self-adjoint Fredholm index} = 0.$$

$$\mathrm{Fred}_0^{sym}(D, H ; T_{ref}) = \{ T = T_{ref} + K \mid K \text{ cpt self-adjoint operator on } H \}$$

strictly speaking it should be $K \circ i : D \to H$, where $i : D \to H$ inclusion as a cpt operator

This space is stratified, $k \in \mathbb{N}$, via

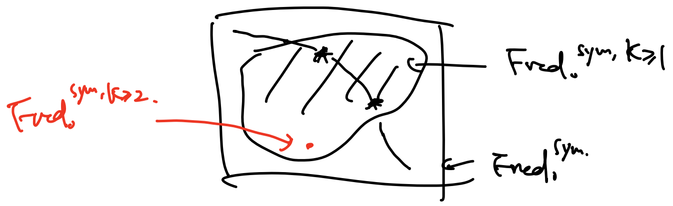
$$\mathrm{Fred}_0^{sym, k}(D, H ; T_{ref}) = \{ T \in \mathrm{Fred}_0^{sym} \mid \ker(\dim T) = k \}.$$

← in fact, a Banach submfd.

Prop 3.27 in [Wen] $\Rightarrow$

$$\mathrm{codim}\, \mathrm{Fred}_0^{sym, k}(D, H ; T_{ref}) = \frac{k(k+1)}{2} \quad (\text{over } \mathbb{R})$$

In particular, for $k \geq 2$, codim ≥ 3. Therefore, for a generic path and its



homotopies, they will only intersect $\mathrm{Fred}_0^{sym,1}$ and avoid $\mathrm{Fred}_0^{sym,k\geq 2}$.

$\Rightarrow$ define for $T=\{T(s)\}_{s\in[-1,1]}\subset \mathrm{Fred}_0^{sym}$ that connects $\underset{T(-1)}{\underset{\shortparallel}{T_-}}$ and $\underset{T(1)}{\underset{\shortparallel}{T_+}} \in \mathrm{Fred}_0^{sym}$,

$\mu^{spec}(T)$ = signed counting of path T intersecting $\mathrm{Fred}_0^{sym,1}$.

<u>Rmk</u> The hard part of computing μ^{spec} is the multiplicities of eigenvalues.
In dim-2, there is a faster way via winding numbers (Section 3.5 in [Wen])